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Hybrid Parameters

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INTRODUCTION

In order to predict the behaviour of a small-signal transistor amplifier, it is important to know its operating characteristics *e.g.*, input impedance, output impedance, voltage gain *etc.* In the text so far, these characteristics were determined by using β and circuit resistance values. This method of analysis has two principal advantages. Firstly, the values of circuit components are readily available and secondly the procedure followed is easily understood. However, the major drawback of this method is that accurate results cannot be obtained. It is because the input and output circuits of a transistor amplifier are not completely independent. For example, output current is affected by the value of load resistance rather than being constant at the value βI_b . Similarly, output voltage has an effect on the input circuit so that changes in the output cause changes in the input.

* Since transistor is generally connected in CE arrangement, current amplification factor β is mentioned here.

One of the methods that takes into account all the effects in a transistor amplifier is the hybrid parameter approach. In this method, four parameters (one measured in ohm, one in mho, two dimensionless) of a transistor are measured experimentally. These are called hybrid or h parameters of the transistor. Once these parameters for a transistor are known, formulas can be developed for input impedance, voltage gain etc. in terms of h parameters. There are two main reasons for using h parameter method in describing the characteristics of a transistor. Firstly, it yields exact results because the inter-effects of input and output circuits are taken into account. Secondly, these parameters can be measured very easily. To begin with, we shall apply h parameter approach to general circuits and then extend it to transistor amplifiers.

24.1 Hybrid Parameters

Every **linear circuit having input and output terminals can be analysed by four parameters (one measured in ohm, one in mho and two dimensionless) called hybrid or h Parameters.*

Hybrid means “mixed”. Since these parameters have mixed dimensions, they are called hybrid parameters. Consider a linear circuit shown in Fig. 24.1. This circuit has input voltage and current labelled v_1 and i_1 . This circuit also has output voltage and current labelled v_2 and i_2 . Note that both input and output currents (i_1 and i_2) are assumed to flow *into* the box ; input and output voltages (v_1 and v_2) are assumed *positive* from the upper to the lower terminals. These are standard conventions and do not necessarily correspond to the actual directions and polarities. When we analyse circuits in which the voltages are of opposite polarity or where the currents flow out of the box, we simply treat these voltages and currents as negative quantities.

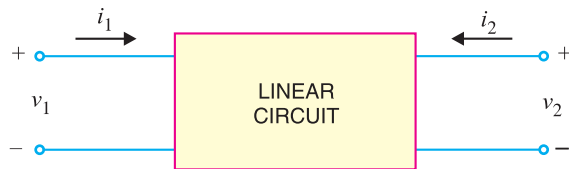


Fig. 24.1

It can be proved by advanced circuit theory that voltages and currents in Fig. 24.1 can be related by the following sets of equations :

$$v_1 = h_{11} i_1 + h_{12} v_2 \quad \dots(i)$$

$$i_2 = h_{21} i_1 + h_{22} v_2 \quad \dots(ii)$$

In these equations, the h s are fixed constants for a given circuit and are called h parameters. Once these parameters are known, we can use equations (i) and (ii) to find the voltages and currents in the circuit. If we look at eq.(i), it is clear that **** h_{11} has the dimension of ohm and h_{12} is dimensionless. Similarly, from eq. (ii), h_{21} is dimensionless and h_{22} has the dimension of mho. The following points may be noted about h parameters :

(i) Every linear circuit has four h parameters ; one having dimension of ohm, one having dimension of mho and two dimensionless.

(ii) The h parameters of a given circuit are constant. If we change the circuit, h parameters would also change.

(iii) Suppose that in a particular linear circuit, voltages and currents are related as under:

$$v_1 = 10i_1 + 6v_2$$

$$i_2 = 4i_1 + 3v_2$$

* A linear circuit is one in which resistances, inductances and capacitances remain fixed when voltage across them changes.

** The two parts on the R.H.S. of eq. (i) must have the unit of voltage. Since current (amperes) must be multiplied by resistance (ohms) to get voltage (volts), h_{11} should have the dimension of resistance *i.e.* ohms.

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Here we can say that the circuit has h parameters given by $h_{11} = 10 \Omega$; $h_{12} = 6$; $h_{21} = 4$ and $h_{22} = 3 \text{ S}$.

24.2 Determination of h Parameters

The major reason for the use of h parameters is the relative ease with which they can be measured. The h parameters of a circuit shown in Fig. 24.1 can be found out as under :

(i) If we short-circuit the output terminals (See Fig. 24.2), we can say that output voltage $v_2 = 0$. Putting $v_2 = 0$ in equations (i) and (ii), we get,

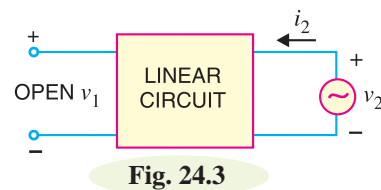
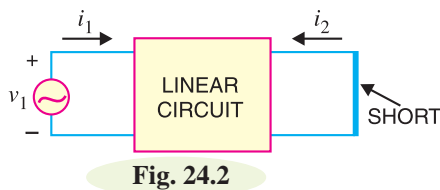
$$v_1 = h_{11} i_1 + h_{12} \times 0$$

$$i_2 = h_{21} i_1 + h_{22} \times 0$$

$$\therefore h_{11} = \frac{v_1}{i_1} \quad \text{for } v_2 = 0 \text{ i.e. output shorted}$$

$$\text{and} \quad h_{21} = \frac{i_2}{i_1} \quad \text{for } v_2 = 0 \text{ i.e. output shorted}$$

Let us now turn to the physical meaning of h_{11} and h_{21} . Since h_{11} is a ratio of voltage and current (i.e. v_1/i_1), it is an impedance and is called ***“input impedance with output shorted”***. Similarly, h_{21} is the ratio of output and input current (i.e., i_2/i_1), it will be dimensionless and is called **“current gain with output shorted”**.



(ii) The other two h parameters (viz h_{12} and h_{22}) can be found by making $i_1 = 0$. This can be done by the arrangement shown in Fig. 24.3. Here, we drive the output terminals with voltage v_2 , keeping the input terminals open. With this set up, $i_1 = 0$ and the equations become :

$$v_1 = h_{11} \times 0 + h_{12} v_2$$

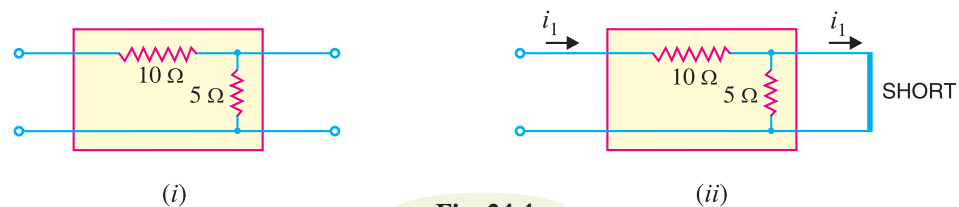
$$i_2 = h_{21} \times 0 + h_{22} v_2$$

$$\therefore h_{12} = \frac{v_1}{v_2} \quad \text{for } i_1 = 0 \text{ i.e. input open}$$

$$\text{and} \quad h_{22} = \frac{i_2}{v_2} \quad \text{for } i_1 = 0 \text{ i.e. input open}$$

Since h_{12} is a ratio of input and output voltages (i.e. v_1/v_2), it is dimensionless and is called **“voltage feedback ratio with input terminals open”**. Similarly, h_{22} is a ratio of output current and output voltage (i.e. i_2/v_2), it will be admittance and is called **“output admittance with input terminals open”**.

Example. 24.1. Find the h parameters of the circuit shown in Fig. 24.4 (i).



* Note that v_1 is the input voltage and i_1 is the input current. Hence v_1/i_1 is given the name input impedance.

Solution. The h parameters of the circuit shown in Fig. 24.4 (i) can be found as under :

To find h_{11} and h_{21} , short - circuit the output terminals as shown in Fig. 24.4 (ii). It is clear that input impedance of the circuit is 10Ω because 5Ω resistance is shorted out.

$$\therefore h_{11} = 10 \Omega$$

Now current i_1 flowing into the box will flow through 10Ω resistor and then through the shorted path as shown. It may be noted that in our discussion, i_2 is the output current flowing into the box. Since output current in Fig. 24.4 (ii) is actually flowing out of the box, i_2 is negative *i.e.*,

$$i_2 = -i_1$$

$$\therefore h_{21} = \frac{i_2}{i_1} = \frac{-i_1}{i_1} = -1$$

To find h_{12} and h_{22} , make the arrangement as shown in Fig. 24.4 (iii). Here we are driving the output terminals with a voltage v_2 . This sets up a current i_2 . Note that input terminals are open. Under this condition, there will be no current in 10Ω resistor and, therefore, there can be no voltage drop across it. Consequently, all the voltage appears across input terminals *i.e.*

$$v_1 = v_2$$

$$\therefore h_{12} = \frac{v_1}{v_2} = \frac{v_2}{v_2} = 1$$

Now the output impedance looking into the output terminals with input terminals open is simply 5Ω . Then h_{22} will be the reciprocal of it because h_{22} is the output admittance with input terminals open.

$$\therefore h_{22} = 1/5 = 0.2 \text{ S}$$

The h parameters of the circuit are :

$$h_{11} = 10 \Omega ; h_{21} = -1$$

$$h_{12} = 1 ; h_{22} = 0.2 \text{ S}$$

It may be mentioned here that in practice, dimensions are not written with h parameters. It is because it is understood that h_{11} is always in ohms, h_{12} and h_{21} are dimensionless and h_{22} is in mhos.

Example 24.2. Find the h parameters of the circuit shown in Fig. 24.5 (i).

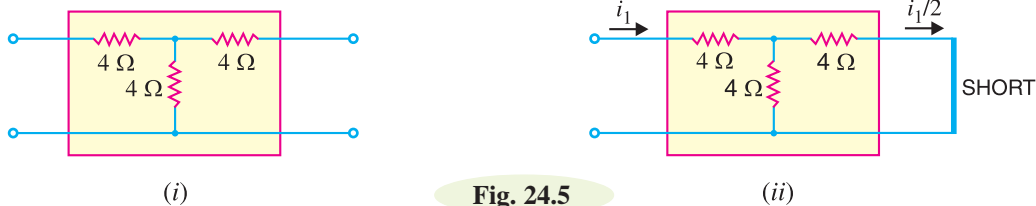


Fig. 24.5

Solution. First of all imagine that output terminals are short-circuited as shown in Fig. 24.5 (ii). The input impedance under this condition is the parameter h_{11} .

$$\text{Obviously, } h_{11} = 4 + 4 \parallel 4$$

$$= 4 + \frac{4 \times 4}{4 + 4} = 6 \Omega$$

Now the input current i_1 in Fig. 24.5 (ii) will divide equally at the junction of 4Ω resistors so that output current is $i_1/2$ *i.e.*

$$i_2 = -i_1/2 = -0.5 i_1$$

$$\therefore h_{21} = \frac{i_2}{i_1} = \frac{-0.5 i_1}{i_1} = -0.5$$

In order to find h_{12} and h_{22} , imagine the arrangement as shown in Fig. 24.5 (iii). Here we are driving the output terminals with voltage v_2 , keeping the input terminals open. Under this condition, any voltage v_2 applied to the output will divide by a factor 2 *i.e.*

$$v_1 = \frac{v_2}{2} = 0.5 v_2$$

$$\therefore h_{12} = \frac{v_1}{v_2} = \frac{0.5 v_2}{v_2} = 0.5$$

Now the output impedance looking into the output terminals with input terminals open is simply 8Ω . Then h_{22} will be the reciprocal of this *i.e.*

$$h_{22} = \frac{1}{8} = 0.125 \text{ } \Omega$$

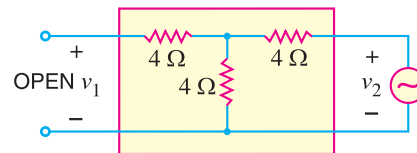


Fig. 24.5 (iii)

24.3 *h* Parameter Equivalent Circuit

Fig. 24.6 (i) shows a linear circuit. It is required to draw the *h* parameter equivalent circuit of Fig. 24.6 (i). We know that voltages and currents of the circuit in Fig. 24.6 (i) can be expressed in terms of *h* parameters as under :

$$v_1 = h_{11} i_1 + h_{12} v_2 \quad \dots(i)$$

$$i_2 = h_{21} i_1 + h_{22} v_2 \quad \dots(ii)$$

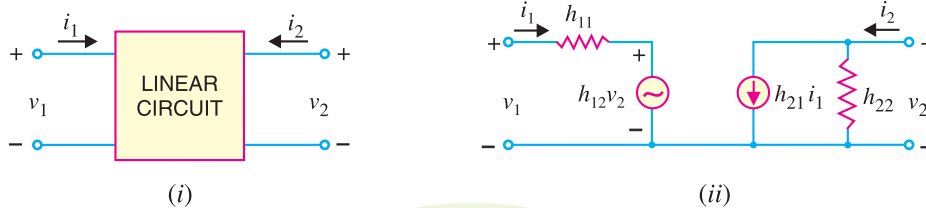


Fig. 24.6

Fig. 24.6 (ii) shows *h* parameter equivalent circuit of Fig. 24.6 (i) and is derived from equations (i) and (ii). The *input circuit* appears as a resistance h_{11} in series with a voltage generator $h_{12} v_2$. This circuit is derived from equation (i). The *output circuit* involves two components ; a current generator $h_{21} i_1$ and shunt resistance h_{22} and is derived from equation (ii). The following points are worth noting about the *h* parameter equivalent circuit [See Fig. 24.6 (ii)] :

(i) This circuit is called hybrid equivalent because its input portion is a Thevenin equivalent, or voltage generator with series resistance, while output side is Norton equivalent, or current generator with shunt resistance. Thus it is a mixture or a hybrid. The symbol '*h*' is simply the abbreviation of the word hybrid (hybrid means "mixed").

(ii) The different hybrid parameters are distinguished by different number subscripts. The notation shown in Fig. 24.6 (ii) is used in general circuit analysis. The first number designates the circuit in which the effect takes place and the second number designates the circuit from which the effect comes. For instance, h_{21} is the "short-circuit forward current gain" or the ratio of the current in the output (circuit 2) to the current in the input (circuit 1).

(iii) The equivalent circuit of Fig. 24.6 (ii) is extremely useful for two main reasons. First, it isolates the input and output circuits, their interaction being accounted for by the two controlled sources. Thus, the effect of output upon input is represented by the equivalent voltage generator $h_{12}v_2$ and its value depends upon output voltage. Similarly, the effect of input upon output is represented by current generator $h_{21}i_1$ and its value depends upon input current. Secondly, the two parts of the circuit are in a form which makes it simple to take into account source and load circuits.

24.4 Performance of a Linear Circuit in h Parameters

We have already seen that any linear circuit with input and output has a set of h parameters. We shall now develop formulas for input impedance, current gain, voltage gain etc. of a linear circuit in terms of h parameters.

(i) **Input impedance.** Consider a linear circuit with a load resistance r_L across its terminals as shown in Fig. 24.7. The input impedance Z_{in} of this circuit is the ratio of input voltage to input current *i.e.*

$$Z_{in} = \frac{v_1}{i_1}$$

Now $v_1 = h_{11}i_1 + h_{12}v_2$ in terms of h parameters. Substituting the value of v_1 in the above expression, we get,

$$Z_{in} = \frac{h_{11}i_1 + h_{12}v_2}{i_1} = h_{11} + \frac{h_{12}v_2}{i_1} \quad \dots(i)$$

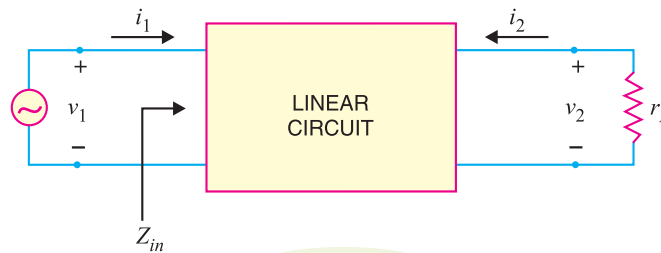


Fig. 24.7

Now, $i_2 = h_{21}i_1 + h_{22}v_2$ in terms of h parameters. Further from Fig. 24.7, it is clear that $i_2 = -v_2/r_L$. The minus sign is used here because the actual load current is opposite to the direction of i_2 .

$$\therefore \frac{-v_2}{r_L} = h_{21}i_1 + h_{22}v_2 \quad \left[\because i_2 = \frac{-v_2}{r_L} \right]$$

$$\text{or} \quad -h_{21}i_1 = h_{22}v_2 + \frac{v_2}{r_L} = v_2 \left(h_{22} + \frac{1}{r_L} \right)$$

$$\therefore \frac{v_2}{i_1} = \frac{-h_{21}}{h_{22} + \frac{1}{r_L}} \quad \dots(ii)$$

Substituting the value of v_2/i_1 from exp. (ii) into exp. (i), we get,

$$Z_{in} = h_{11} - \frac{h_{12}h_{21}}{h_{22} + \frac{1}{r_L}} \quad \dots(iii)$$

This is the expression for input impedance of a linear circuit in terms of h parameters and load connected to the output terminals. If either h_{12} or r_L is very small, the second term in exp. (iii) can be neglected and input impedance becomes :

$$Z_{in} \simeq h_{11}$$

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(ii) Current Gain. Referring to Fig. 24.7, the current gain A_i of the circuit is given by :

$$A_i = \frac{i_2}{i_1}$$

Now

$$i_2 = h_{21} i_1 + h_{22} v_2$$

and

$$v_2 = -i_2 r_L$$

∴

$$i_2 = h_{21} i_1 - h_{22} i_2 r_L$$

or

$$i_2 (1 + h_{22} r_L) = h_{21} i_1$$

or

$$\frac{i_2}{i_1} = \frac{h_{21}}{1 + h_{22} r_L}$$

But $i_2/i_1 = A_i$, the current gain of the circuit.

∴

$$A_i = \frac{h_{21}}{1 + h_{22} r_L}$$

If $h_{22} r_L \ll 1$, then $A_i \approx h_{21}$.

The expression $A_i \approx h_{21}$ is often useful. To say that $h_{22} r_L \ll 1$ is the same as saying that $r_L \ll 1/h_{22}$. This occurs when r_L is much smaller than the output resistance ($1/h_{22}$), shunting $h_{21} i_1$ generator. Under such condition, most of the generator current bypasses the circuit output resistance in favour of r_L . This means that $i_2 \approx h_{21} i_1$ or $i_2/i_1 \approx h_{21}$.

(iii) Voltage gain. Referring back to Fig. 24.7, the voltage gain of the circuit is given by :

$$\begin{aligned} A_v &= \frac{v_2}{v_1} \\ &= \frac{v_2}{i_1 Z_{in}} \quad (\because v_1 = i_1 Z_{in}) \end{aligned} \quad \dots(iv)$$

While developing expression for input impedance, we found that :

$$\frac{v_2}{i_1} = \frac{-h_{21}}{h_{22} + \frac{1}{r_L}}$$

Substituting the value of v_2/i_1 in exp. (iv), we get,

$$A_v = \frac{-h_{21}}{Z_{in} \left(h_{22} + \frac{1}{r_L} \right)}$$

(iv) Output impedance. In order to find the output impedance, remove the load r_L , set the signal voltage v_1 to zero and connect a generator of voltage v_2 at the output terminals. Then h parameter equivalent circuit becomes as shown in Fig. 24.8. By definition, the output impedance Z_{out} is

$$Z_{out} = \frac{v_2}{i_2}$$

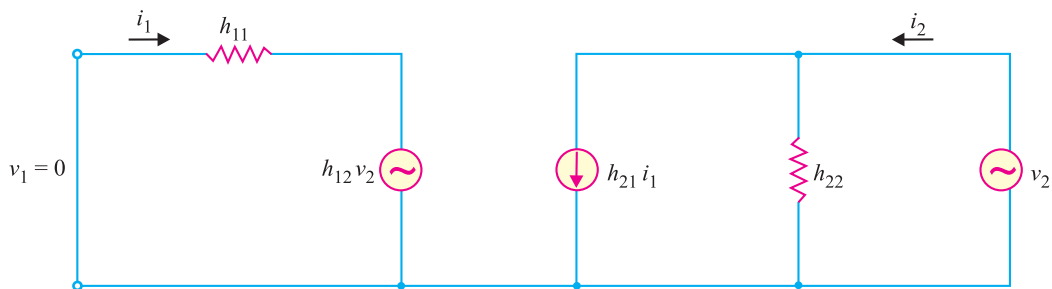


Fig. 24.8

With $v_1 = 0$ and applying Kirchhoff's voltage law to the input circuit, we have,

$$0 = i_1 h_{11} + h_{12} v_2$$

$$\therefore i_1 = -\frac{h_{12} v_2}{h_{11}}$$

Now
$$i_2 = h_{21} i_1 + h_{22} v_2$$

Putting the value of $i_1 (= -h_{12} v_2/h_{11})$ in this equation, we get,

$$i_2 = h_{21} \left(-\frac{h_{12} v_2}{h_{11}} \right) + h_{22} v_2$$

or
$$i_2 = -\frac{h_{21} h_{12} v_2}{h_{11}} + h_{22} v_2$$

Dividing throughout by v_2 , we have,

$$\frac{i_2}{v_2} = \frac{-h_{21} h_{12}}{h_{11}} + h_{22}$$

$$\therefore Z_{out} = \frac{v_2}{i_2} = \frac{1}{h_{22} - \frac{h_{21} h_{12}}{h_{11}}}$$

Example 24.3. Find the (i) input impedance and (ii) voltage gain for the circuit shown in Fig. 24.9.

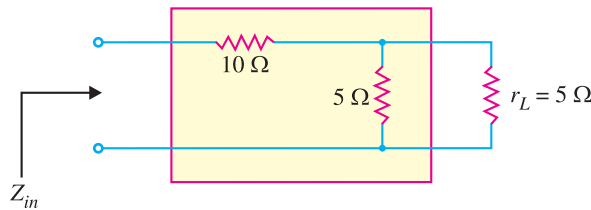


Fig. 24.9

Solution. The h parameters of the circuit inside the box are the same as those calculated in example 24.1 *i.e.*

$$h_{11} = 10; \quad h_{21} = -1$$

$$h_{12} = 1 \quad \text{and} \quad h_{22} = 0.2$$

(i) Input impedance is given by :

$$\begin{aligned} Z_{in} &= h_{11} - \frac{h_{12} h_{21}}{h_{22} + \frac{1}{r_L}} = 10 - \frac{1 \times -1}{0.2 + \frac{1}{5}} \\ &= 10 + 2.5 = \mathbf{12.5 \Omega} \end{aligned}$$

By inspection, we can see that input impedance is equal to 10Ω plus two 5Ω resistances in parallel *i.e.*

$$\begin{aligned} Z_{in} &= 10 + 5 \parallel 5 \\ &= 10 + \frac{5 \times 5}{5 + 5} = 12.5 \Omega \end{aligned}$$

(ii) Voltage gain,
$$A_v = \frac{-h_{21}}{Z_{in} \left(h_{22} + \frac{1}{r_L} \right)} = \frac{1}{12.5 \left(0.2 + \frac{1}{5} \right)} = \mathbf{\frac{1}{5}}$$

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It means that output voltage is one-fifth of the input voltage. This can be readily established by inspection of Fig. 24.9. The two $5\ \Omega$ resistors in parallel give a net resistance of $2.5\ \Omega$. Therefore, we have a voltage divider consisting of $10\ \Omega$ resistor in series with $2.5\ \Omega$ resistor.

$$\begin{aligned} \therefore \quad \text{Output voltage} &= \frac{2.5}{12.5} \times \text{Input voltage} \\ \text{or} \quad \frac{\text{Output voltage}}{\text{Input voltage}} &= \frac{2.5}{12.5} = \frac{1}{5} \\ \text{or} \quad A_v &= \frac{1}{5} \end{aligned}$$

Comments. The reader may note that in a simple circuit like that of Fig. 24.9, it is not advisable to use h parameters to find the input impedance and voltage gain. It is because answers of such circuits can be found directly by inspection. However, in complicated circuits, inspection method becomes cumbersome and the use of h parameters yields quick results.

24.5 The h Parameters of a Transistor

It has been seen in the previous sections that every linear circuit is associated with h parameters. When this linear circuit is terminated by load r_L , we can find input impedance, current gain, voltage gain, etc. in terms of h parameters. Fortunately, for *small* a.c. signals, the transistor behaves as a linear device because the output a.c. signal is directly proportional to the input a.c. signal. Under such circumstances, the a.c. *operation* of the transistor can be described in terms of h parameters. The expressions derived for input impedance, voltage gain *etc.* in the previous section shall hold good for transistor amplifier except that here r_L is the a.c. load seen by the transistor.

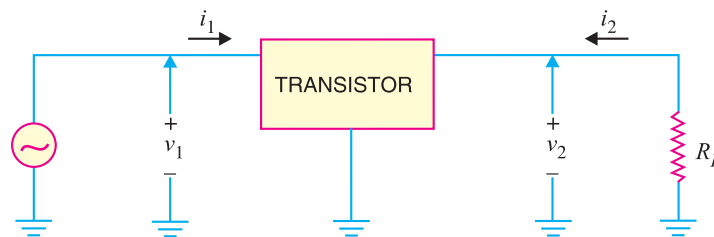


Fig. 24.10

Fig. 24.10 shows the transistor amplifier circuit. There are four quantities required to describe the external behaviour of the transistor amplifier. These are v_1 , i_1 , v_2 and i_2 shown on the diagram of Fig. 24.10. These voltages and currents can be related by the following sets of equations :

$$\begin{aligned} v_1 &= h_{11} i_1 + h_{12} v_2 \\ i_2 &= h_{21} i_1 + h_{22} v_2 \end{aligned}$$

The following points are worth noting while considering the behaviour of transistor in terms of h parameters :

(i) For small a.c. signals, a transistor behaves as a linear circuit. Therefore, its a.c. operation can be described in terms of h parameters.

(ii) The value of h parameters of a transistor will depend upon the transistor connection (*i.e.* CB , CE or CC) used. For instance, a transistor used in CB arrangement may have $h_{11} = 20\ \Omega$. If we use the same transistor in CE arrangement, h_{11} will have a different value. Same is the case with other h parameters.

(iii) The expressions for input impedance, voltage gain *etc.* derived in Art. 24.4 are also applicable to transistor amplifier except that r_L is the a.c. load seen by the transistor *i.e.*

$$r_L = R_C \parallel R_L$$

(iv) The values of h parameters depend upon the operating point. If the operating point is changed, parameter values are also changed.

(v) The notations v_1, i_1, v_2 and i_2 are used for general circuit analysis. In a transistor amplifier, we use the notation depending upon the configuration in which transistor is used. Thus for CE arrangement,

$$v_1 = V_{be} \ ; \ i_1 = I_b \ ; \ v_2 = V_{ce} \ ; \ i_2 = I_c$$

Here V_{be}, I_b, V_{ce} and I_c are the R.M.S. values.

24.6 Nomenclature for Transistor h Parameters

The numerical subscript notation for h parameters (*viz.* h_{11}, h_{21}, h_{12} and h_{22}) is used in general circuit analysis. However, this nomenclature has been modified for a transistor to indicate the nature of parameter and the transistor configuration used. The h parameters of a transistor are represented by the following notation :

- (i) The numerical subscripts are replaced by letter subscripts.
- (ii) The first letter in the double subscript notation indicates the nature of parameter.
- (iii) The second letter in the double subscript notation indicates the circuit arrangement (*i.e.* CB, CE or CC) used.

Table below shows the h parameter nomenclature of a transistor :

S.No.	h parameter	Notation in CB	Notation in CE	Notation in CC
1.	h_{11}	h_{ib}	h_{ie}	h_{ic}
2.	h_{12}	h_{rb}	h_{re}	h_{rc}
3.	h_{21}	h_{fb}	h_{fe}	h_{fc}
4.	h_{22}	h_{ob}	h_{oe}	h_{oc}

Note that first letter i, r, f or o indicates the nature of parameter. Thus h_{11} indicates input impedance and this parameter is designated by the subscript i . Similarly, letters r, f and o respectively indicate reverse voltage feedback ratio, forward current transfer ratio and output admittance. The second letters b, e and c respectively indicate CB, CE and CC arrangement.

24.7 Transistor Circuit Performance in h Parameters

The expressions for input impedance, voltage gain etc. in terms of h parameters derived in Art. 24.4 for general circuit analysis apply equally for transistor analysis. However, it is profitable to rewrite them in standard transistor h parameter nomenclature.

(i) **Input impedance.** The general expression for input impedance is

$$Z_{in} = h_{11} - \frac{h_{12} h_{21}}{h_{22} + \frac{1}{r_L}}$$

Using standard h parameter nomenclature for transistor, its value for CE arrangement will be :

$$Z_{in} = h_{ie} - \frac{h_{re} h_{fe}}{h_{oe} + \frac{1}{r_L}}$$

Similarly, expressions for input impedance in CB and CC arrangements can be written. It may be noted that r_L is the a.c. load seen by the transistor.

(ii) **Current gain.** The general expression for current gain is

$$A_i = \frac{h_{21}}{1 + h_{22} r_L}$$

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Using standard transistor h parameter nomenclature, its value for CE arrangement is

$$A_i = \frac{h_{fe}}{1 + h_{oe} r_L}$$

The reader can readily write down the expressions for CB and CC arrangements.

(iii) Voltage gain. The general expression for voltage gain is

$$A_v = \frac{-h_{21}}{Z_{in} \left(h_{22} + \frac{1}{r_L} \right)}$$

Using standard transistor h parameter nomenclature, its value for CE arrangement is

$$A_v = \frac{-h_{fe}}{Z_{in} \left(h_{oe} + \frac{1}{r_L} \right)}$$

In the same way, expressions for voltage gain in CB and CC arrangement can be written.

(iv) Output impedance. The general expression for output impedance is

$$Z_{out} = \frac{1}{h_{22} - \frac{h_{21} h_{12}}{h_{11}}}$$

Using standard transistors h parameter nomenclature, its value for CE arrangement is

$$Z_{out} = \frac{1}{h_{oe} - \frac{h_{fe} h_{re}}{h_{ie}}}$$

In the same way, expression for output impedance in CB and CC arrangements can be written.

The above expression for Z_{out} is for the transistor. If the transistor is connected in a circuit to form a single stage amplifier, then output impedance of the stage = $Z_{out} \parallel r_L$ where $r_L = R_C \parallel R_L$.

Example 24.4. A transistor used in CE arrangement has the following set of h parameters when the d.c. operating point is $V_{CE} = 10$ volts and $I_C = 1$ mA :

$$h_{ie} = 2000 \Omega; \quad h_{oe} = 10^{-4} \text{ mho}; \quad h_{re} = 10^{-3}; \quad h_{fe} = 50$$

Determine (i) input impedance (ii) current gain and (iii) voltage gain. The a.c. load seen by the transistor is $r_L = 600 \Omega$. What will be approximate values using reasonable approximations?

Solution. (i) Input impedance is given by :

$$\begin{aligned} Z_{in} &= h_{ie} - \frac{h_{re} h_{fe}}{h_{oe} + \frac{1}{r_L}} = 2000 - \frac{10^{-3} \times 50}{10^{-4} + \frac{1}{600}} \quad \dots (i) \\ &= 2000 - 28 = \mathbf{1972 \Omega} \end{aligned}$$

The second term in eq. (i) is quite small as compared to the first.

$$\therefore Z_{in} \simeq h_{ie} = \mathbf{2000 \Omega}$$

$$(ii) \quad \text{Current gain, } A_i = \frac{h_{fe}}{1 + h_{oe} \times r_L} = \frac{50}{1 + (600 \times 10^{-4})} = \mathbf{47}$$

If $h_{oe} r_L \ll 1$, then $A_i \simeq h_{fe} = \mathbf{50}$

$$(iii) \quad \text{Voltage gain, } A_v = \frac{-h_{fe}}{Z_{in} \left(h_{oe} + \frac{1}{r_L} \right)} = \frac{-50}{1972 \left(10^{-4} + \frac{1}{600} \right)} = \mathbf{-14.4}$$

The negative sign indicates that there is 180° phase shift between input and output. The magni-

tude of gain is 14.4. In other words, the output signal is 14.4 times greater than the input and it is 180° out of phase with the input.

Example 24.5. A transistor used in CE connection has the following set of h parameters when the d.c. operating point is $V_{CE} = 5$ volts and $I_C = 1$ mA :

$$h_{ie} = 1700 \Omega; h_{re} = 1.3 \times 10^{-4}; h_{fe} = 38; h_{oe} = 6 \times 10^{-6} \text{ } \mathfrak{U}$$

If the a.c. load r_L seen by the transistor is 2 k Ω , find (i) the input impedance (ii) current gain and (iii) voltage gain.

Solution. (i) The input impedance looking into the base of transistor is

$$Z_{in} = h_{ie} - \frac{h_{re} h_{fe}}{h_{oe} + \frac{1}{r_L}} = 1700 - \frac{1.3 \times 10^{-4} \times 38}{6 \times 10^{-6} + \frac{1}{2000}} \approx 1690 \Omega$$

(ii) Current gain, $A_i = \frac{h_{fe}}{1 + h_{oe} r_L} = \frac{38}{1 + 6 \times 10^{-6} \times 2000} = \frac{38}{1.012} \approx 37.6$

(iii) Voltage gain, $A_v = \frac{-h_{fe}}{Z_{in} \left(h_{oe} + \frac{1}{r_L} \right)} = \frac{-38}{1690 \left(6 \times 10^{-6} + \frac{1}{2000} \right)} = 44.4$

Example 24.6. Fig. 24.11 shows the transistor amplifier in CE arrangement. The h parameters of transistor are as under :

$$h_{ie} = 1500 \Omega; h_{fe} = 50; h_{re} = 4 \times 10^{-4}; h_{oe} = 5 \times 10^{-5} \text{ } \mathfrak{U}$$

Find (i) a.c. input impedance of the amplifier (ii) voltage gain and (iii) output impedance.

Solution. The a.c. load r_L seen by the transistor is equivalent of the parallel combination of R_C ($= 10$ k Ω) and R_L ($= 30$ k Ω) i.e.

$$r_L = \frac{R_C R_L}{R_C + R_L} = \frac{10 \times 30}{10 + 30} = 7.5 \text{ k}\Omega = 7500 \Omega$$

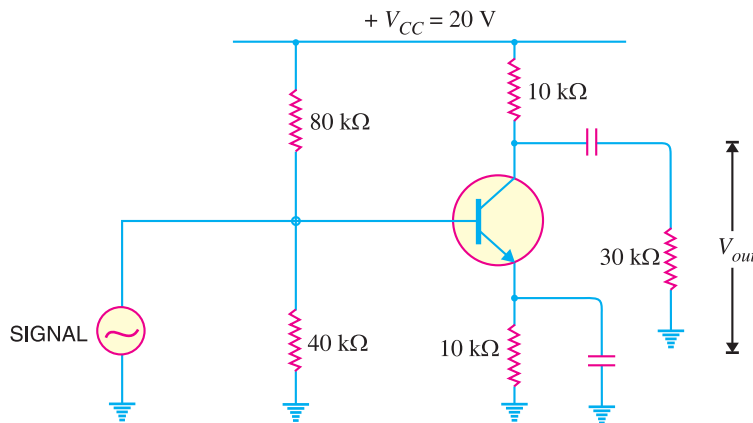


Fig. 24.11

(i) The input impedance looking into the base of transistor is given by :

$$Z_{in} = h_{ie} - \frac{h_{re} h_{fe}}{h_{oe} + \frac{1}{r_L}} = 1500 - \frac{4 \times 10^{-4} \times 50}{5 \times 10^{-5} + \frac{1}{7500}} = 1390 \Omega$$

This is only the input impedance looking into the base of transistor. The a.c. input impedance of the entire stage will be Z_{in} in parallel with bias resistors i.e.

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Input impedance of stage = $80 \times 10^3 \parallel 40 \times 10^3 \parallel 1390 = \mathbf{1320 \Omega}$

(ii) Voltage gain, $A_v = \frac{-h_{fe}}{Z_{in} \left(h_{oe} + \frac{1}{r_L} \right)} = \frac{-50}{1390 \left(5 \times 10^{-5} + \frac{1}{7500} \right)} = \mathbf{-196}$

The negative sign indicates phase reversal. The magnitude of gain is 196.

(iii) Output impedance of transistor is

$$Z_{out} = \frac{1}{h_{oe} - \frac{h_{fe} h_{re}}{h_{ie}}}$$

$$= \frac{1}{5 \times 10^{-5} - \frac{50 \times 4 \times 10^{-4}}{1500}} = 27270 \Omega = 27.27 \text{ k}\Omega$$

∴ Output impedance of the stage

$$= Z_{out} \parallel R_L \parallel R_C$$

$$= 27.27 \text{ k}\Omega \parallel 30 \text{ k}\Omega \parallel 10 \text{ k}\Omega = \mathbf{5.88 \text{ k}\Omega}$$

Example 24.7. Find the value of current gain for the circuit shown in Fig. 24.12. The *h*-parameter values of the transistor are given alongside the figure.

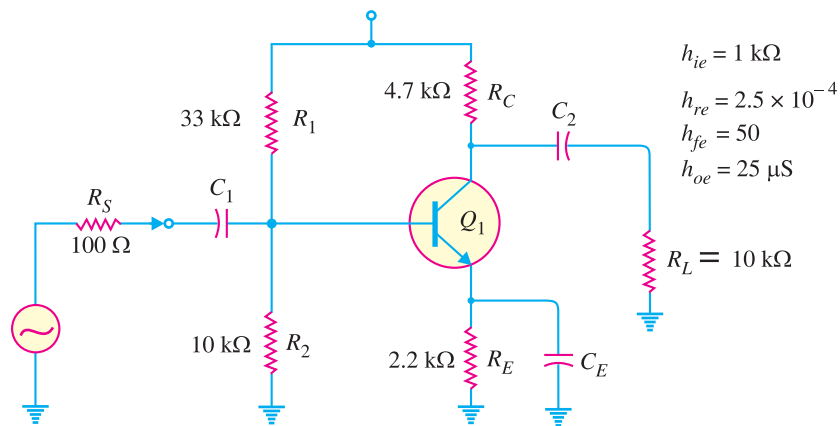


Fig. 24.12

Solution. The current gain A_i for the circuit is given by ;

$$A_i = \frac{h_{fe}}{1 + h_{oe} r_L}$$

Here $r_L = R_C \parallel R_L = 4.7 \text{ k}\Omega \parallel 10 \text{ k}\Omega = 3.2 \text{ k}\Omega$

∴
$$A_i = \frac{50}{1 + (25 \times 10^{-6}) (3.2 \times 10^3)} = \mathbf{46.3}$$

Note that current gain of the circuit is very close to the value h_{fe} . The reason for this is that $h_{oe} r_L \ll 1$. Since this is normally the case, $A_i \approx h_{fe}$.

Example 24.8. In the above example, determine the output impedance of the transistor.

Solution. Note that the signal source (See Fig. 24.12) has resistance $R_S = 100 \Omega$.

∴ Output impedance Z_{out} of the transistor is

$$\begin{aligned}
 Z_{out} &= \frac{1}{h_{oe} - \left(\frac{h_{fe} h_{re}}{h_{ie} + R_S} \right)} \\
 &= \frac{1}{(25 \times 10^{-6}) - \frac{(50)(2.5 \times 10^{-4})}{(1 \times 10^3) + 100}} = 73.3 \times 10^3 \Omega = \mathbf{73.3 \text{ k}\Omega}
 \end{aligned}$$

24.8 Approximate Hybrid Formulas for Transistor Amplifier

The h -parameter formulas (CE configuration) covered in Art. 24.7 can be approximated to a form that is easier to handle. While these approximate formulas will not give results that are as accurate as the original formulas, they can be used for many applications.

(i) Input impedance

$$\text{Input impedance, } Z_{in} = h_{ie} - \frac{h_{re} h_{fe}}{h_{oe} + \frac{1}{r_L}}$$

In actual practice, the second term in this expression is very small as compared to the first term.

$$\therefore Z_{in} = h_{ie} \quad \dots \text{ approximate formula}$$

(ii) Current gain

$$\text{Current gain, } A_i = \frac{h_{fe}}{1 + h_{oe} r_L}$$

In actual practice, $h_{oe} r_L$ is very small as compared to 1.

$$\therefore A_i = h_{fe} \quad \dots \text{ approximate formula}$$

(iii) Voltage gain

$$\begin{aligned}
 \text{Voltage gain, } A_v &= \frac{-h_{fe}}{Z_{in} \left(h_{oe} + \frac{1}{r_L} \right)} \\
 &= \frac{-h_{fe} r_L}{Z_{in} (h_{oe} r_L + 1)}
 \end{aligned}$$

Now approximate formula for Z_{in} is h_{ie} . Also $h_{oe} r_L$ is very small as compared to 1.

$$\therefore A_v = -\frac{h_{fe} r_L}{h_{ie}} \quad \dots \text{ approximate formula}$$

(iv) Output impedance

$$\text{Output impedance of transistor, } Z_{out} = \frac{1}{h_{oe} - \frac{h_{fe} h_{re}}{h_{ie}}}$$

The second term in the denominator is very small as compared to h_{oe} .

$$\therefore Z_{out} = \frac{1}{h_{oe}} \quad \dots \text{ approximate formula}$$

The output impedance of transistor amplifier

$$= Z_{out} \parallel r_L \quad \text{where } *r_L = R_C \parallel R_L$$

* If the amplifier is unloaded (i.e. $R_L = \infty$), $r_L = R_C$.

Example 24.9. For the circuit shown in Fig. 24.13, use approximate hybrid formulas to determine (i) the input impedance (ii) voltage gain. The h parameters of the transistor are $h_{ie} = 1.94 \text{ k}\Omega$ and $h_{fe} = 71$.

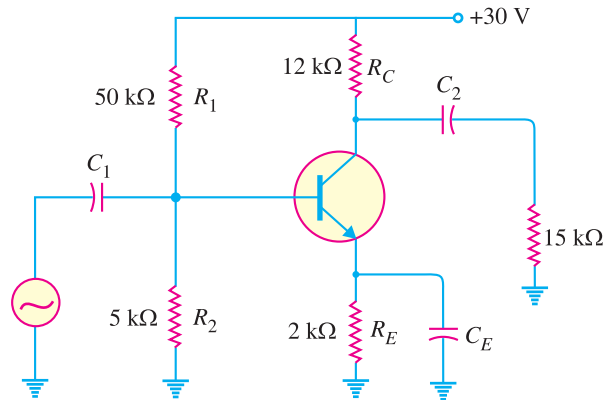


Fig. 24.13

Solution.

$$\text{a.c. collector load, } r_L = R_C \parallel R_L = 12 \text{ k}\Omega \parallel 15 \text{ k}\Omega = 6.67 \text{ k}\Omega$$

(i) Transistor input impedance is

$$Z_{in(\text{base})} = h_{ie} = 1.94 \text{ k}\Omega$$

$$\begin{aligned} \therefore \text{Circuit input impedance} &= Z_{in(\text{base})} \parallel R_1 \parallel R_2 \\ &= 1.94 \text{ k}\Omega \parallel 50 \text{ k}\Omega \parallel 5 \text{ k}\Omega = \mathbf{1.35 \text{ k}\Omega} \end{aligned}$$

$$(ii) \quad \text{Voltage gain, } A_v = \frac{h_{fe} r_L}{h_{ie}} = \frac{71 \times 6.67 \text{ k}\Omega}{1.94 \text{ k}\Omega} = \mathbf{244}$$

Example 24.10. A transistor used in an amplifier has h -parameter values of $h_{ie} = 600 \Omega$ to 800Ω and $h_{fe} = 110$ to 140 . Using approximate hybrid formula, determine the voltage gain for the circuit. The a.c. collector load, $r_L = 460 \Omega$.

Solution. When *minimum* and *maximum* h -parameter values are given, we should determine the *geometric average* of the two values. Thus the values that we would use in the analysis of circuit are found as under :

$$\begin{aligned} h_{ie} &= \sqrt{h_{ie(\text{min})} \times h_{ie(\text{max})}} \\ &= \sqrt{(600 \Omega) (800 \Omega)} = 693 \Omega \end{aligned}$$

$$\begin{aligned} h_{fe} &= \sqrt{h_{fe(\text{min})} \times h_{fe(\text{max})}} \\ &= \sqrt{(110) (140)} = 124 \end{aligned}$$

$$\text{Voltage gain, } A_v = \frac{h_{fe} r_L}{h_{ie}} = \frac{(124) (460)}{693} = \mathbf{82.3}$$

24.9 Experimental Determination of Transistor h Parameters

The determination of h parameters of a general linear circuit has already been discussed in Art. 24.2. To illustrate how such a procedure is carried out for a *CE* transistor amplifier, consider the circuit of Fig. 24.14. The R.M.S. values will be considered in the discussion. Using standard transistor nomenclature :

$$V_{be} = h_{ie} I_b + h_{re} V_{ce} \quad \dots(i)$$

$$I_c = h_{fe} I_b + h_{oe} V_{ce} \quad \dots(ii)$$

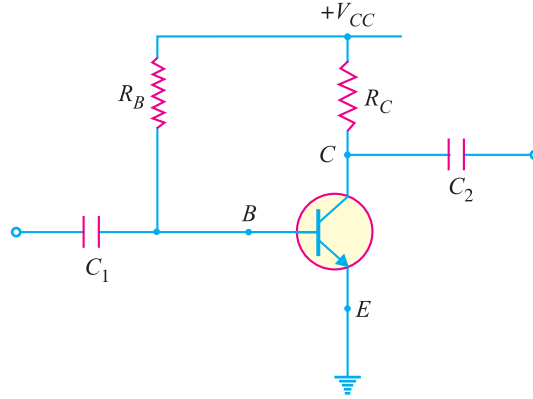


Fig. 24.14

(i) **Determination of h_{fe} and h_{ie} .** In order to determine these parameters, the output is a.c. short circuited as shown in Fig. 24.15 (i). This is accomplished by making the capacitance of C_2 deliberately large. The result is that changing component of collector current flows through C_2 instead of R_C and a.c. voltage developed across C_2 is zero i.e. $*V_{ce} = 0$.

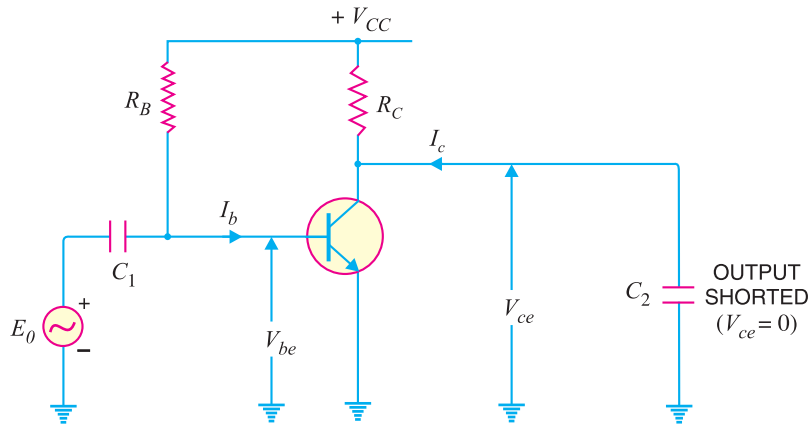


Fig. 24.15 (i)

Substituting $V_{ce} = 0$ in equations (i) and (ii) above, we get,

$$V_{be} = h_{ie} I_b + h_{re} \times 0$$

$$I_c = h_{fe} I_b + h_{oe} \times 0$$

$$\therefore h_{fe} = \frac{I_c}{I_b} \text{ for } V_{ce} = 0$$

$$\text{and } h_{ie} = \frac{V_{be}}{I_b} \text{ for } V_{ce} = 0$$

Note that I_c and I_b are the a.c. R.M.S. collector and base currents respectively. Also V_{be} is the a.c. R.M.S. base-emitter voltage.

* Note that setting $V_{ce} = 0$ does not mean that V_{CE} (the d.c. collector-emitter voltage) is zero. Only a.c. output is short-circuited.

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(ii) **Determination of h_{re} and h_{oe} .** In order to determine these two parameters, the input is a.c. open-circuited, a signal generator is applied across the output and resulting V_{be} , V_{ce} and I_c are measured. This is illustrated in Fig 24.15 (ii). A large inductor L is connected in series with R_B . Since the d.c. resistance of inductor is very small, it does not disturb the operating point. Again, a.c. current cannot flow through R_B because of large reactance of inductor. Further, the voltmeter used to measure V_{be} has a high input impedance and hence there are no paths connected to the base with any appreciable a.c. current. This means that base is *effectively a.c. open-circuited i.e. $I_b = 0$.

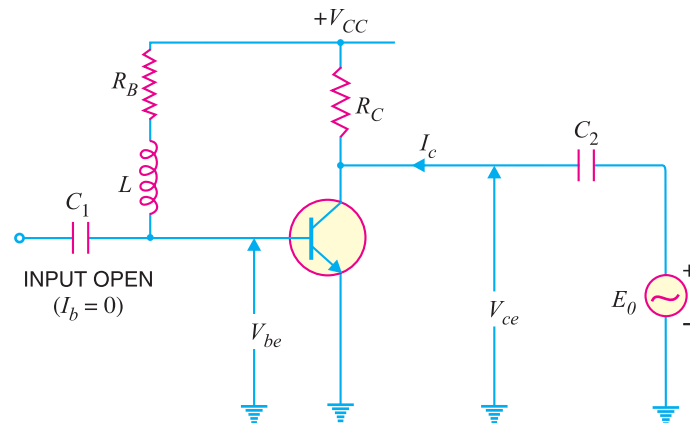


Fig. 24.15 (ii)

Substituting $I_b = 0$ in equations (i) and (ii), we get,

$$V_{be} = h_{ie} \times 0 + h_{re} V_{ce}$$

$$I_c = h_{fe} \times 0 + h_{oe} V_{ce}$$

$$\therefore h_{re} = \frac{V_{be}}{V_{ce}} \text{ for } I_b = 0$$

and
$$h_{oe} = \frac{I_c}{V_{ce}} \text{ for } I_b = 0$$

Example 24.11. The following quantities are measured in a CE amplifier circuit :

(a) With output a.c. short-circuited (i.e. $V_{ce} = 0$)

$$I_b = 10 \mu\text{A}; I_c = 1 \text{ mA}; V_{be} = 10 \text{ mV}$$

(b) With input a.c. open-circuited (i.e. $I_b = 0$)

$$V_{be} = 0.65 \text{ mV}; I_c = 60 \mu\text{A}; V_{ce} = 1 \text{ V}$$

Determine all the four h parameters.

Solution.
$$h_{ie} = \frac{V_{be}}{I_b} = \frac{10 \times 10^{-3}}{10 \times 10^{-6}} = 1000 \Omega$$

$$h_{fe} = \frac{I_c}{I_b} = \frac{1 \times 10^{-3}}{10 \times 10^{-6}} = 100$$

$$h_{re} = \frac{V_{be}}{V_{ce}} = \frac{0.65 \times 10^{-3}}{1} = 0.65 \times 10^{-3}$$

* How effectively the base is a.c. open-circuited depends upon the reactance L and the input impedance of the voltmeter used to measure V_{be} .

$$h_{oe} = \frac{I_c}{V_{ce}} = \frac{60 \times 10^{-6}}{1} = 60 \mu\text{mho}$$

24.10 Limitations of h Parameters

The h parameter approach provides accurate information regarding the current gain, voltage gain, input impedance and output impedance of a transistor amplifier. However, there are two major limitations on the use of these parameters.

(i) It is very difficult to get the exact values of h parameters for a particular transistor. It is because these parameters are subject to considerable variation—unit to unit variation, variation due to change in temperature and variation due to change in operating point. In predicting an amplifier performance, care must be taken to use h parameter values that are correct for the operating point being considered.

(ii) The h parameter approach gives correct answers for small a.c. signals only. It is because a transistor behaves as a linear device for small signals only.

MULTIPLE-CHOICE QUESTIONS

- Hybrid means
 - mixed
 - single
 - unique
 - none of the above
- There are h parameters of a transistor.
 - two
 - four
 - three
 - none of the above
- The h parameter approach gives correct results for
 - large signals only
 - small signals only
 - both small and large signals
 - none of the above
- A transistor behaves as a linear device for
 - small signals only
 - large signals only
 - both small and large signals
 - none of the above
- The parameter h_{ie} stands for input impedance in
 - CB arrangement with output shorted
 - CC arrangement with output shorted
 - CE arrangement with output shorted
 - none of the above
- The dimensions of h_{ie} parameter are
 - mho
 - ohm
 - farad
 - none of the above
- The parameter h_{fe} is called in CE arrangement with output shorted.
 - voltage gain
 - current gain
 - input impedance
 - none of the above
- If the operating point changes, the h parameters of a transistor
 - also change
 - do not change
 - may or may not change
 - none of the above
- The values of h parameter of a transistor in CE arrangement are arrangement.
 - the same as for CB
 - the same as for CC
 - different from that in CB
 - none of the above
- In order to determine h_{fe} and h_{ie} parameters of a transistor, is a.c. short-circuited.
 - input
 - output
 - input as well as output
 - none of the above
- If temperature changes, h parameters of a transistor
 - may or may not change
 - do not change
 - also change
 - none of the above
- In CE arrangement, the value of input impedance is approximately equal to

- (i) h_{ie} (ii) h_{oe}
 (iii) h_{re} (iv) none of the above
13. Using standard transistor h parameter nomenclature, the voltage gain in CE arrangement is
- (i) $\frac{-h_{fe}}{Z_{in} \left(h_{oe} + \frac{1}{r_L} \right)}$
 (ii) $\frac{-h_{fe}}{Z_{out} (h_{oe} + 1)}$
- (iii) $\frac{-h_{fe}}{h_{oe} + h_{re}}$
 (iv) none of the above
14. $Z_{in} = h_{ie} - \frac{\dots}{h_{oe} + \frac{1}{r_L}}$
- (i) $h_{re} h_{oe}$ (ii) $h_{re} h_{fe}$
 (iii) $r_L h_{oe}$ (iv) none of the above
15. h parameters of a transistor are dimensionless.
- (i) four (ii) three
 (iii) two (iv) none of the above

Answers to Multiple-Choice Questions				
1. (i)	2. (ii)	3. (ii)	4. (i)	5. (iii)
6. (ii)	7. (ii)	8. (i)	9. (iii)	10. (ii)
11. (iii)	12. (i)	13. (i)	14. (ii)	15. (iii)

Chapter Review Topics

1. What do you understand by hybrid parameters ? What are their dimensions ?
2. How will you measure h parameters of a linear circuit ?
3. Draw the h parameter equivalent circuit of a linear circuit.
4. What is the physical meaning of h parameters ?
5. Derive the general formula for :
 (i) input impedance (ii) current gain and (iii) voltage gain in terms of h parameters and the load.
6. What are the notations for h parameters of a transistor when used in (i) CB (ii) CE and (iii) CC arrangement ?
7. How are h parameters of a transistor measured ?
8. What are the drawbacks of h parameter approach in the design of a transistor amplifier ?

Problems

1. Determine the h -parameter values for the circuit shown in Fig. 24.16.

$[h_{11} = 8\Omega ; h_{21} = -0.5 ; h_{12} = 0.5 ; h_{22} = 0.125 \text{ } \Omega^{-1}]$

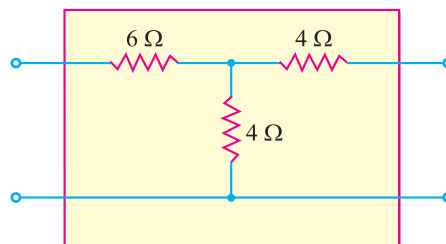


Fig. 24.16

2. Measurement of a circuit gives the following h -parameters :

$h_{11} = 10 \text{ k}\Omega ; h_{12} = 0.5 ; h_{21} = 100 ; h_{22} = 2 \text{ mS}$

Find v_1 and i_2 if $i_1 = 1 \text{ mA}$ and $v_2 = 2\text{V}$.

$[v_1 = 11\text{V} ; i_2 = 14 \text{ mA}]$

3. A CE amplifier has $h_{ie} = 1000\Omega$, $h_{re} = 10^{-4}$, $h_{fe} = 100$ and $h_{oe} = 12 \times 10^{-6}\text{S}$. The load resistance = 2000Ω . Find (i) current gain (ii) voltage gain (iii) output resistance.
 [(i) 97.7 (ii) – 199.2 (iii) $5 \times 10^5\Omega$]
4. A CE amplifier has $h_{ie} = 1500\Omega$, $h_{fe} = -60$ and $h_{oe} = 12.5 \times 10^{-6}\text{S}$. The load resistance varies between $5 \times 10^3\Omega$ and $10 \times 10^3\Omega$. Find the maximum and minimum values of (i) current gain (ii) voltage gain.
 [(i) 36.9, 26.7 (ii) 178, 123]
5. An amplifier has values of $R_C = 12\text{ k}\Omega$, $R_L = 4.7\text{ k}\Omega$, $R_1 = 33\text{ k}\Omega$, $R_2 = 4.7\text{ k}\Omega$ and $I_C = 1\text{ mA}$. At 1 mA, the transistor has h -parameter values of $h_{ie} = 1\text{ k}\Omega$ to $5\text{ k}\Omega$ and $h_{fe} = 70$ to 350. Determine the values of (i) input impedance (ii) voltage gain for the circuit. Use approximate hybrid formulas.
 [(i) 1.45 k Ω (ii) 236.9]

Discussion Questions

1. What is the origin of the name hybrid ?
2. How can we obtain an effective a.c. short-circuit across the output of an amplifier ? Does this affect d.c. operating conditions ?
3. When h parameters are specified for a particular transistor, the operating point is usually given. Why is this necessary ?
4. How can we obtain an effective a.c. open circuit at the input to an amplifier ? Does this affect d.c. operating conditions ?
5. Under what condition is the input impedance of a transistor equal to h_{ie} ?